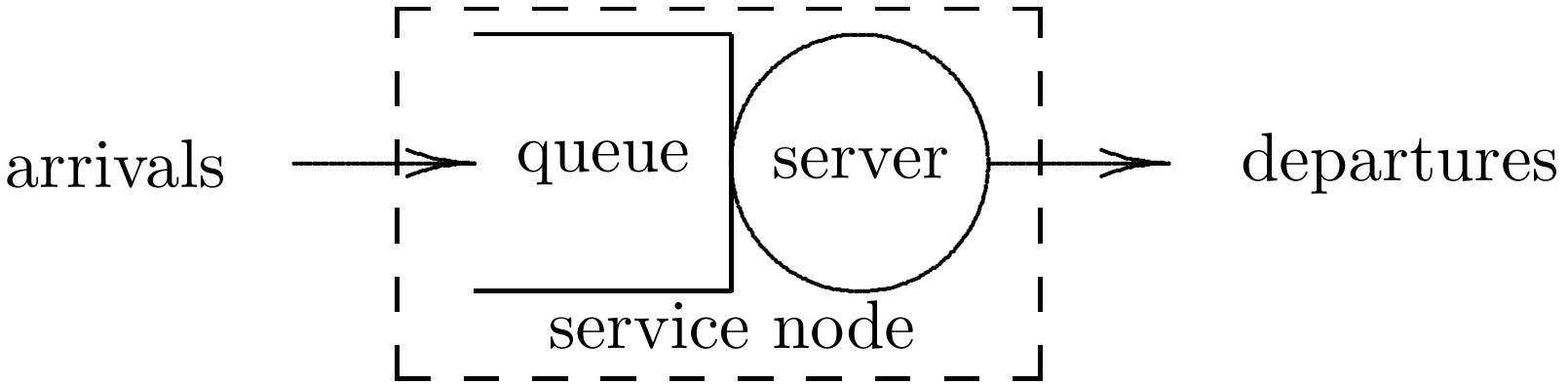
Section 1.2: A Single-Server Queue



- A *single-sever service node* consists of a server plus its queue
- If there is only one service technician, the machine shop model from section 1.1 is a single-server queue

Queue Discipline

Queue discipline: the algorithm used when a job is selected from the queue to enter service

- *FIFO* – first in, first out
- *LIFO* – last in, first out
- *SIRO* – serve in random order
- *Priority* – typically shortest job first (SJF)

Assumptions

- FIFO is also known as first come, first serve (FCFS)
  - The order of arrival and departure are the same
  - This observation can be used to simplify the simulation
  - Unless otherwise specified, assume FIFO with infinite queue capacity.
- Service is *non-preemptive*
  - Once initiated, service of a job will continue until completion
- Service is *conservative*
  - Server will never remain idle if there is one or more jobs in the service node

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a

FIFO SSQ Example

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

What is the average time *in the queue* per job?

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

What is the average time *in the queue* per job? $\frac{(7-4)+(8-5)+(10-8)}{6}$

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	in queue	in service	departed
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

What is the average # of jobs *in the queue* per unit time?

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

What is the average # of jobs *in the queue* per unit time? $\frac{1(5-4)+2(7-5)+1(8-7)+1(10-8)}{18}$

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

How would the results differ if we dequeued a **random** job?

a_i is **arrival time**
s_i is **service time**

job	a_i	s_i
a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

FIFO SSQ Example

How would the results differ if we dequeued a **random** job? The service orders shown are one scenario, since $t = 7$ is the only time with > 1 job in the queue. If at $t = 7$ job **c** were chosen instead of **b**, when job **c** completed ($t = 9$) we would have both **b** and **d** in the queue...

a_i is **arrival time**
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a	3	4
b	4	1
c	5	2
d	8	2
e	13	1
f	14	4

t	**in queue**	**in service**	**departed**
1	—	—	—
2	—	—	—
3	—	a	—
4	b	a	—
5	c, b	a	—
7	c	b	a
8	d	c	b, a
10	—	d	c, b, a
12	—	—	d, c, b, a
13	—	e	d, c, b, a
14	—	f	e, d, c, b, a
18	—	—	f, e, d, c, b, a

SSQ in the Reading

- What types of queue disciplines are there (FIFO, random, ???)
- What types of SSQs is most of the text focused on? What other properties do these SSQs have?
- Read about **job**-averaged SSQ measures
- Read about **time**-averaged SSQ measures
- Read how these two types of SSQ measures are connected to each other