# A Simple Monte Carlo Experiment

October 9, 2025

# Simple Monte Carlo Experiment

**Conceptual Model**
We have an arbitrarily large urn of marbles labeled with distinct values in $[0, N-1]$ (only 1 marble with each number). We would like to estimate the value of $N$ (and not by inspecting every single marble).

**Specification Model** We can estimate $N$ by drawing $z$ marbles ($x_i$s, **with replacement**) and calculating their average value $\bar{x}$. We expect this to be near the true average, $\mu$, which can be written using Gauss' Law

$$\bar{x} = \frac{1}{z}\sum_{i=1}^{z} x_i \approx \mu = \frac{1}{N}\sum_{i=0}^{N-1} i = \frac{1}{N}\left(\frac{(N-1)(N)}{2}\right) = \frac{N-1}{2} \quad \rightarrow \quad N \approx 1 + \frac{2}{z}\sum_{i=1}^{z} x_i$$

We are **given** $N$, we will simulate the above sample and averaging procedure arriving at a **simulated estimate** for $N$ — if our simulation can be validated, we can use this approach in the real world!

# Simple Monte Carlo Experiment

**Conceptual Model**
We have an arbitrarily large urn of marbles labeled with distinct values in $[0, N-1]$ (only 1 marble with each number). We would like to estimate the value of $N$ (and not by inspecting every single marble).
**Specification Model** We can estimate $N$ by drawing $z$ marbles ($x_i$s, **with replacement**) and calculating their average value $\bar{x}$. We expect this to be near the true average, $\mu$, which can be written using Gauss' Law

$$\bar{x} = \frac{1}{z}\sum_{i=1}^{z} x_i \approx \mu = \frac{1}{N}\sum_{i=0}^{N-1} i = \frac{1}{N}\left(\frac{(N-1)(N)}{2}\right) = \frac{N-1}{2} \quad \rightarrow \quad N \approx 1 + \frac{2}{z}\sum_{i=1}^{z} x_i$$

We are **given** $N$, we will simulate the above sample and averaging procedure arriving at a **simulated estimate** for $N$ — if our simulation can be validated, we can use this approach in the real world!
2 minutes — Your language library doesn't have a $Random()$ providing $u \in (0,1)$, instead it has `RandomInteger()` with with range $[0,$ `RAND_MAX`$]$.
**How would your group design a computational model?**

# Simple Monte Carlo Experiment

## Computational Model

Program `simple-monte-carlo.c` takes as input $N$ and calculates the averages from small samplings ($z = \lfloor 0.15N \rfloor$, $\{x_i\}_{i=1}^{z}$) of integral values on $[0, N-1]$.

We use a pRNG library with a `RandomInteger()` function that returns values within $[0, \texttt{RAND\_MAX}]$, $\texttt{RAND\_MAX} > N$.

We simulate the drawing and replacement of a labeled marble $x_i$ with

$$x_i \leftarrow \texttt{RandomInteger()} \mod \texttt{N}$$

We calculate the average of each $k$ sample and track how many $\bar{x}_k < \mu$, and how many $\bar{x}_k \geq \mu$ in a `counts[2]` array.

*"…view the source, Luke!"*

# Simple Monte Carlo Experiment

## Verification

For verification, we'll simply observe if many $\bar{x}_k$s fall symmetrically about the known, true $\mu$.

# Simple Monte Carlo Experiment

## Verification

For verification, we'll simply observe if many $\bar{x}_k$s fall symmetrically about the known, true $\mu$.

***Demonstrate...***

```
# n not a variable, it's a command; N=10000, implicit sample size z=0.15N, SEED?
$ ./simple-monte-carlo n 10000 SEED
```

# Simple Monte Carlo Experiment

## Verification

For verification, we'll simply observe if many $\bar{x}_k$s fall symmetrically about the known, true $\mu$.

***Demonstrate...***

```
# n not a variable, it's a command; N=10000, implicit sample size z=0.15N, SEED?
$ ./simple-monte-carlo n 10000 SEED
```

Adding a fourth command line parameter allows many "samples" to be drawn, each with their own $\bar{x}_k$ estimate of $\mu$. In simulation speak, we call these ***replications*** ($n$). In this case intermediate console reports show:

- a particular $k^{\text{th}}$ experiment's results,
- the total number of sample $\bar{x}_k$s that have been to the left and right of $\mu$

```
# N=10000, SEED?, replications=100
$ ./simple-monte-carlo n 10000 SEED 100
```

If we collect many samples ($k = 1, \ldots, B$) for a single experiment, each with an $\bar{x}_k$, how do we expect this set of $\{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k, \ldots, \bar{x}_B\}$ to be distributed around the true $\mu$?

*drumroll please*

# Oops

Clearly

`RandomInteger()` mod `N`

doesn't perform quite as advertised for some `N`s.

**Why?** We'll answer this later in lecture, first math (yay!)

# `equilikely-monte-carlo n 1717600 SEED 1000`

The author says to select random array elements with `a[ Equilikely(0,n-1)]`...

This sounds a lot like drawing $[0, N-1]$ labeled marbles out of an urn...

Could it be that $Equilikely(a, b)$ fixes our `simple-monte-carlo` problems?
***Demonstration***...

# `equilikely-monte-carlo n 1717600 SEED 1000`

The author says to select random array elements with `a[ Equilikely(0,n-1)]`...

This sounds a lot like drawing $[0, N-1]$ labeled marbles out of an urn...

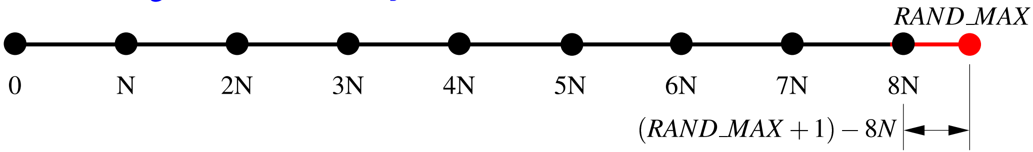
Could it be that $Equilikely(a,b)$ fixes our `simple-monte-carlo` problems?
***Demonstration***...

But `./equilikely-monte-carlo` working doesn't explain why

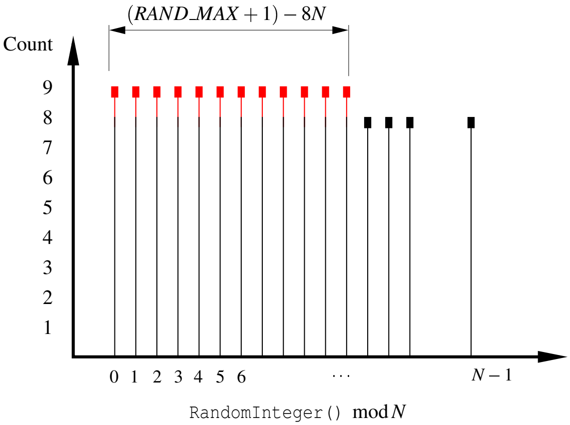
$$x_i \leftarrow \texttt{RandomInteger()} \mod \texttt{N}$$

was wrong!

## Range of `RandomInteger()` Partitioned into $N$ Sized Chunks




## Data Histogram of `RandomInteger()` mod $N$




The red residual of the `RandomInteger()` range contributes disproportionally to the smaller values of the

`RandomInteger()` mod $N$

histogram.

Depending on `RAND_MAX` and $N$, and the use case of results, this *might* be negligible.

If it is not, drastically wrong results can occur.

`RAND_MAX`, the largest value returned by `RandomInteger()` (not always ρ), is largely independent of the underlying pRNG. It is tied to the machine+software architecture!
So choosing a "better" pRNG doesn't make this technique OK!

# $Equilikely(a,b)$: A Solution to `RandomInteger()` mod $N$...

Is there a **safe range** to use the simple

`RandomInteger()` mod $N$

technique for random integral values?

Who cares! — just use the correct algorithm (`Equilikely(a,b)`)

1. Independent of the pRNG used, its period ($\rho$), and the architecture (`int` size)
2. Independent of `N`
3. You still have to generate one random number (no savings there)
4. Proper `Random()` functions return $u \in (0,1)$ already, **not integers**
5. “Technically correct” is the best kind of correct `:)`

Why care? — Consider the canonical **Fisher-Yates** shuffling algorithm, web examples for which almost always use the `RandomInteger()` mod $N$ technique for choosing the next array element. While this technique may be sufficient for small array sizes, this demonstration suggests **it does not scale** to large arrays. “Big Data” anyone?