Uniform Arrivals not Uniform Interarrivals

October 7, 2025

LGT Topics to Discuss

Q1 $a_i \sim$ sorted $Uniform(0, 1000)$ **Q3**: a_i accumulated from $r_i \sim Uniform(0, 2)$

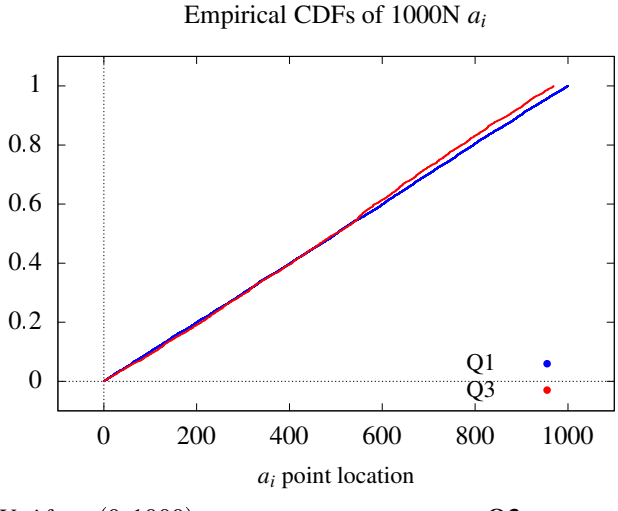
Questions 1 and 3: Suppose these were the arrival times for an **SSQ**, if the $\{a_*\}$ of the two questions have close to identical distributions, does it matter **how** they were constructed?
Come up with **two different** reasons why the answer is **YES**...

Q2 $r_i \sim Exponential(\bar{r})$ **Q4** r_i from sorted $a_i - a_{i-1}$

Questions 2 and 4: Arrival times for the SSQ simulations were generated in two different ways. Do your SSQ results for these two methods differ or appear the same?
In other words, did the method of generating arrival times (a_i) effect the SSQ results?

One person in your group: clearly document your groups findings, add your group name, and send them to me via Email (cc the members of your group) before the end of LGT.

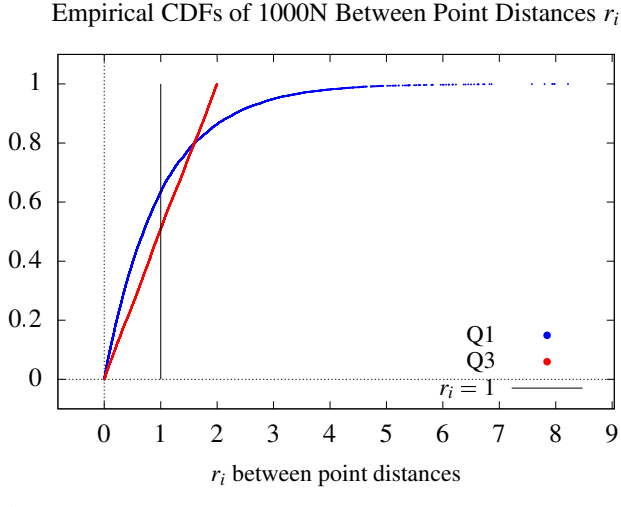
(my graphs)



Q1: $a_i \sim$ sorted $Uniform(0, 1000)$

Q3: a_i accumulated from $r_i \sim Uniform(0, 2)$

(my graphs)



Q1: r_i from sorted $a_i - a_{i-1}$

Q3: $r_i \sim Uniform(0,2)$

Drum Roll Please. . . (your thoughts?)

Questions 1 and 3: Suppose these were the arrival times for an **SSQ**, if the $\{a_*\}$ of the two questions have close to identical distributions, does it matter **how** they were constructed?

Drum Roll Please. . . (your thoughts?)

Questions 1 and 3: Suppose these were the arrival times for an **SSQ**, if the $\{a_*\}$ of the two questions have close to identical distributions, does it matter **how** they were constructed?

Yes - It is computationally more efficient to generate a new a_{i+1} when we need, instead of generating them all up front and sorting a huge array.
Have you sorted two billion `double`'s lately?

Yes - We expect the **different distributions** of the $\{r_*\}$ will have an effect on at least one measure: either traffic intensity, or $\bar{x}$, or $\bar{q}$, or . . .

Yes - *Validation!* - we expect only one of the $\{r_*\}$ distributions to be true for a particular system.

Drum Roll Please. . . (your thoughts?)

Questions 2 and 4: Arrival times for the SSQ simulations were generated in two different ways. Do your SSQ results for these two methods differ or appear the same?
No difference (in theory).

Hopefully, we'll have time in lecture to prove this later in the course. . .

Two Different Arrival Models

Uniformly distributed arrival times

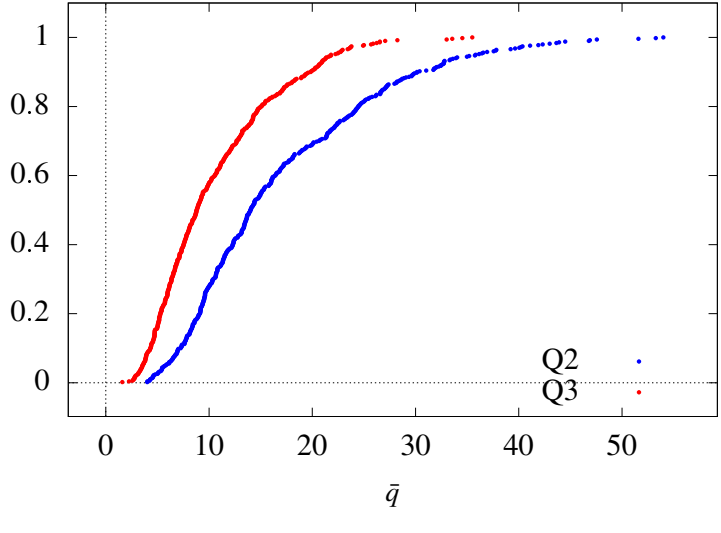
Q1 a_i sorted from many $Uniform(0, \tau)$ OR **Q2** $r_i \sim Exponential(\bar{r})$

vs

Interarrival times that are uniformly distributed

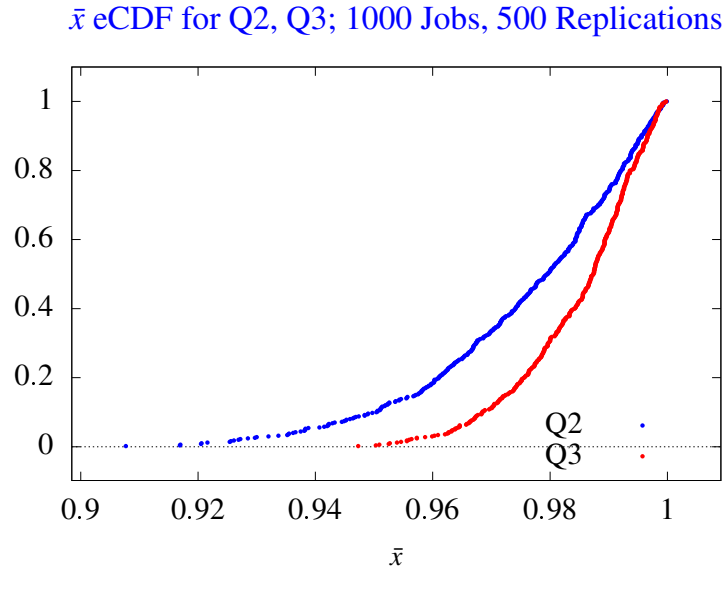
Q3 $r_i \sim Uniform(0, 2\bar{r})$

$\bar{q}$ eCDF for Q2, Q3; 1000 Jobs, 500 Replications



Q2: $r_i \sim Exponential(1)$

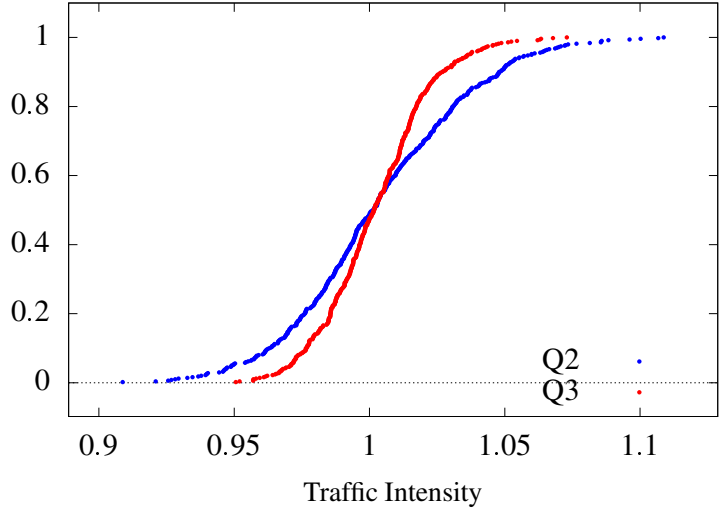
Q3: $r_i \sim Uniform(0,2)$



Q2: $r_i \sim Exponential(1)$

Q3: $r_i \sim Uniform(0,2)$

Traffic Intensity eCDF for Q2, Q3; 1000 Jobs, 500 Reps



Q2: $r_i \sim Exponential(1)$

Q3: $r_i \sim Uniform(0,2)$

Two Different Arrival Models

Uniformly distributed arrival times

Q1 a_i sorted from many $Uniform(0, \tau)$ OR **Q2** $r_i \sim Exponential(\bar{r})$

not the same as

Interarrival times that are uniformly distributed

Q3 $r_i \sim Uniform(0, 2\bar{r})$

$\bar{q}$ and $\bar{x}$ **Q3** CDFs from $r_i \sim Uniform(0, 2\bar{r})$ appear shifted away from **Q2** CDFs.

It takes **only one differently distributed metric** to veto the idea that the two implementations are **simulating the same system.**

Two Different Arrival Models

Uniformly distributed arrival times

Q1 a_i sorted from many $Uniform(0, \tau)$ OR **Q2** $r_i \sim Exponential(\bar{r})$

not the same as

Interarrival times that are uniformly distributed

Q3 $r_i \sim Uniform(0, 2\bar{r})$

Which is **more common** in the ℜeal world?

Two Different Arrival Models

In the absense of contradicting data or theoretical explanations, exponential interarrivals are the first "best guess" of the simulation writer.

For performance reasons, generate a_i from

$$r_i \sim Exponential(\bar{r}) = Exponential\left(\frac{a_n}{n}\right)$$

The connection between uniformly distributed a_i and exponentially distributed r_i will be shown (yay! a proof!) in the second half of the semester.