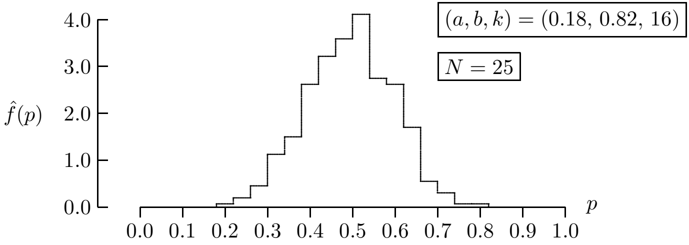
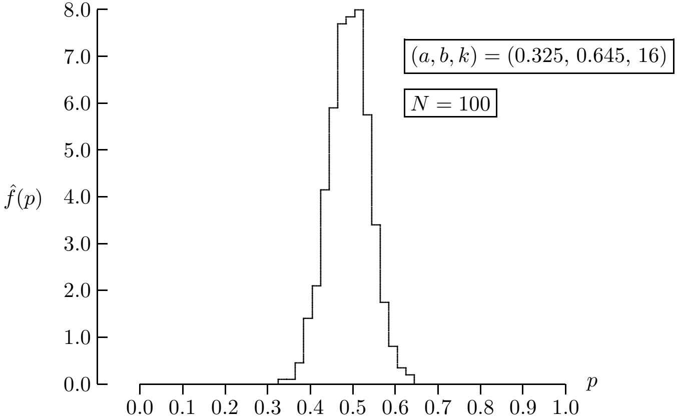
Let’s Gamble!

What is the probability of winning at the game of *craps*?

- $n = 1000$ estimates of craps for $N = 25$ plays



- Note these are *density* estimates, not *relative frequency* estimates
- As $N \to \infty$, histogram will become taller and narrower
- Centered on mean, consistent with $\int_0^1 \hat{f}(p)\, dp = 1$



- Four-fold increase in N yields two-fold decrease in uncertainty

By increasing from N to $4N$ the authors show a halving of s?

Does this make sense? You take N Monte Carlo experiments measuring x_i and you have

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

Going to $4N$ means you have …

$$s_{4N}^2 = \frac{1}{4N}\left\{\sum_{i=1}^{N}(x_i - \bar{x})^2 + \sum_{i=N+1}^{2N}(x_i - \bar{x})^2 + \sum_{i=2N+1}^{3N}(x_i - \bar{x})^2 + \sum_{i=3N+1}^{4N}(x_i - \bar{x})^2\right\}$$

$$s_{4N}^2 \approx \frac{4 \cdot Ns^2}{4N} = s^2$$

…each one of those addends will be **about** the same number, namely $N\sigma^2$ where σ is the true, naturally occuring, “spread” in the phenomenon.
Distributions cannot be narrowed by running more experiments, if so validation needs to be revisited!

What is the mistake in this interpretation of the authors’ statements?

By increasing from N to $4N$ the authors show a halving of s?

What is the mistake in this interpretation of the authors' statements?

There are two:

1. The algebra is correct, but the "suspect" s derivation shown is not the sample deviation that is changing (how could it, algebraically it shouldn't).

   Each replication (there are $n = 1000$ of them) contributes one estimate of the true $p = Pr(\text{Winning Craps})$. The law of large numbers (LLN) says that as the number of trials (N) increases, the average moves closer to the true p.
   This hinted at in a previous slide as $\lim_{N \to \infty} \bar{x}$ is a single value.

   The s that is decreasing comes from

$$s^2 = \frac{1}{n} \sum_n (t_j - \bar{t})^2 \qquad t_j = \bar{x}_j = \frac{1}{N} \sum_N x_i$$

   where j indexes over the 1000 replications, t_j is the point estimate of p from trial j, x_is in each trial is either 1 or 0 for a win or loss of one game.
   The deviation of an individual replication or trial is uncalculated and unused.
   s is decreasing because each point estimate x_i is closer to p due to LLN.

2.

By increasing from N to $4N$ the authors show a halving of s?

What is the mistake in this interpretation of the authors' statements?

There are two:

1.
2.
   - The probability of winning the dice game *craps* is a **number, not a distribution**. We can arrive at this value through analytical techniques (possible with *craps*, not all games).
     Or we can run a Monte Carlo simulation to estimate the value. By choosing simulation we also choose to accept a margin of error in our results (the complement of our "Confidence Level").

   - The author's aren't talking about decreasing the "spread" in a naturally occuring distribution (in general you *can't*) . They are talking about *increasing* the confidence of a simulation measurement (which is to say $\bar{t}$ is within a particular interval).

What implication does the “square root rule” have for a career in developing computer simulation experiements?