First Sets of Non-Terminals N

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3	$S \rightarrow H\ \$$
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Write a sentence from the language of this grammar…

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Write a sentence from the language of this grammar...

What terminals of the grammar ($\Sigma = \{b, c, e, g, h\}$) can a sentential form β begin with?[1]

How is this different than the previous question? **We just want the first terminal**, and we are interested in **all possible sentences** of the language.

[1] Recall a **sentential form** is $S \Rightarrow^* \beta$ and $\beta \in (N \cup \Sigma)^*$

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Congratulations (I hope), you've just found (most of)
the **First Set** of S:

$$First(S) \approx \{b, c, e, g, h\}$$

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$First(E) = \{e\}$

Why isn't c in $First(E)$? Notice that $E \rightarrow \lambda$ and rules 2, 7, 9 and 6 permit

$$\begin{array}{rlcl} (2) & S & \rightarrow & E\,F\,G\,H\,\$ \\ (7) & S & \rightarrow & \lambda\,F\,G\,H\,\$ \\ (9) & S & \rightarrow & C\,E\,G\,H\,\$ \\ (6\&7) & S & \rightarrow & c\,\lambda\,G\,H\,\$ \\ & S & \rightarrow & c\,G\,H\,\$ \end{array}$$

so can't c begin an E?

It's time for a formal definition of $First(\alpha)$...

First Sets of α (Formal Definition)

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$$\texttt{First}(\alpha) = \{t \in \Sigma_{\$} | \alpha \Rightarrow^{*} t\ \beta\} \qquad \alpha \in N, \quad \beta \in (N \bigcup \Sigma)^{*}$$

Notice: we aren't interested in **sentential forms**, ie: all the rules containing α in the RHS. $First(\alpha)$ is just the terminals that can **begin** the RHS of a derivation **with α on the LHS**.

(We also permit ourselves to say $t \Rightarrow t$, even though t is a terminal (or even $\$$) and does not exist on the LHS of any production rule; what is easy about $\alpha \in \Sigma_{\$}$?)

First Sets of α (Computational Definition)

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$$First(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in \Sigma_\$ \\ \displaystyle\bigcup_{(\alpha \rightarrow \mathcal{X}_i \beta_i) \in P} First(\mathcal{X}_i \beta_i) & \text{if } \alpha \in N \end{cases}$$

$$First(\alpha_1 \alpha_2 \cdots \alpha_n) = \bigcup_{j=1}^{n} First(\alpha_j) \text{ if } \alpha_i \Rightarrow^* \lambda \text{ for } i = 1, \ldots, j-1$$

The first set of a terminal is itself, the first set of a non-terminal is the union of the first sets of its production rules' LHSs, and the first set of a sequence is the union of its element's first sets **from left to right** up to and including **the first symbol that cannot derive to λ.**

Notice that first sets are from $\Sigma_\$$, the complete $First(S)$ is actually $\{b, c, e, g, h, \$\}$, because $\{\$\} \subset First(E\,F\,G\,H\,\$)$ and $First(H\,\$) = \{h, \$\}$.

Follow Sets of Non-Terminals N

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What symbols from $\Sigma_\$$ might **follow** the rewrite of $B \rightarrow b$ in a derivation using rule 4?

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What symbols from $\Sigma_\$$ might **follow** the rewrite of $B \rightarrow b$ in a derivation using rule 4?

B is in the RHS of only rule 1 (which makes things simpler), **in this case** the "follow set" of $B \equiv First(C\,\$)$ because $C\,\$$ comes after B in rule 1.

$$Follow(B) = First(C\,\$) = \{c, \$\}$$

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A little more nuanced: what is the follow set of E?

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A little more nuanced: what is the follow set of E?

E is in the RHS of rules 2 and 9,

- from rule 2, $Follow(E)$ gets a contribution from $First(F\,G\,H\,\$) = \{c, e, g\}$
- from rule 9, $Follow(E)$ gets a contribution from $Follow(F)$, since E is at the end of the rewrite rule for F. Fortuitously, the $Follow(F)$ is straightforward in this case: $Follow(F) = First(G\,H\,\$) = \{g\}$

$$Follow(E) = First(F\,G\,H\,\$) \bigcup Follow(F) = \{c, e, g\}$$

Follow Sets of A (Definition)

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$$\texttt{Follow}(A) = \{t \in \Sigma_\$ \mid S \Rightarrow^+ \alpha A\, t\, \beta\}$$

$$A \in N \qquad \alpha, \beta \in (N \cup \Sigma)^*$$

In this case, we are interested in **only sentential forms** of the language.

There is no difference between calculating $Follow(t \in \Sigma)$ vs $Follow(A \in N)$, it turns out we will be interested in only the follow sets of **non-terminals** from the grammar.

Follow Sets of A (Computational Definition)

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$$\texttt{Follow}(A) = \{t \in \Sigma_\$ | S \Rightarrow^+ \alpha A t \beta\}$$

$$A \in N \qquad \alpha, \beta \in (N \cup \Sigma)^*$$

i. Set $Follow(A) = \emptyset$

ii. For each instance of A in a production $X \rightarrow \alpha A \beta$,

   a. Add $First(\beta)$ to $Follow(A)$

   b. If $\beta \Rightarrow^* \lambda$, add $Follow(X)$ to $Follow(A)$