Correct this Grammar to be LL(1)

#	Rules
1	$S \rightarrow A\,B\,\$$
2	$S \rightarrow B\,C\,\$$
3	$A \rightarrow A\,t\,C\,x$
4	$A \rightarrow g$
5	$B \rightarrow y\,A\,B$
6	$B \rightarrow h$
7	$C \rightarrow x\,C\,y$
8	$C \rightarrow p$

The grammar is **not LL(1)** due to the left-recursive rule 3.

Unfortunately, it doesn't fit into our "left factoring pattern:"

$$\begin{array}{lcl} A & \rightarrow & A\gamma\beta \\ A & \rightarrow & \beta \end{array} \quad \Rightarrow \quad \begin{array}{lcl} A & \rightarrow & \beta R \\ R & \rightarrow & \gamma\beta R \\ & | & \lambda \end{array}$$

(γ may be "empty," recall lower Greek letters are $(\Sigma + N)*$)

While we can set $\gamma = t\,C$, β **cannot be both** g **and** x

What to do?

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Changing the A productions to

$$\begin{array}{lcl} A & \rightarrow & g\,A\,t\,C\,x \\ & \mid & \lambda \end{array}$$

permits sentences with too many gs:

$$\begin{array}{l} A \Rightarrow g\,A\,t\,C\,x \\ A \Rightarrow g\,g\,A\,t\,C\,x\,t\,C\,x \\ A \Rightarrow g\,g\,\lambda\,t\,p\,x\,t\,p\,x \end{array}$$

The original grammar permits only one g per A recursion.

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We have to use a different (new) non-terminal on the RHS of the new A rule:

$$\begin{array}{lcl} A & \rightarrow & A\,t\,C\,x \\ A & \rightarrow & g \end{array} \equiv \begin{array}{lcl} A & \rightarrow & g\,Q \\ Q & \rightarrow & Q\,t\,C\,x \\ Q & \rightarrow & \lambda \end{array} \equiv \begin{array}{lcl} A & \rightarrow & g\,Q \\ Q & \rightarrow & t\,C\,x\,Q \\ Q & \rightarrow & \lambda \end{array}$$

This equivilency for Q can be reasoned out with a little bit of thought, but it also falls out of our left-factoring pattern if we bend the rules a smidge and recognize Q can be written as $Q \rightarrow Q\,t\,C\,x\,\lambda$ and letting $\gamma = t\,C\,x$ and $\beta = \lambda$.