Correct this Grammar to be LL(1)

#	Rules
1	$S \rightarrow A\ B\ \$$
2	$S \rightarrow B\ C\ \$$
3	$A \rightarrow A\ t\ C\ x$
4	$A \rightarrow g$
5	$B \rightarrow y\ A\ B$
6	$B \rightarrow h$
7	$C \rightarrow x\ C\ y$
8	$C \rightarrow p$

The grammar is **not LL(1)** due to the left-recursive rule 3.

Unfortunately, it doesn't fit into our "left factoring pattern:"

$$\begin{array}{lcl} A & \rightarrow & A\gamma\beta \\ A & \rightarrow & \beta \end{array} \quad \Rightarrow \quad \begin{array}{lcl} A & \rightarrow & \beta R \\ R & \rightarrow & \gamma\beta R \\ & | & \lambda \end{array}$$

(γ may be "empty," recall lower Greek letters are $(\Sigma + N)*$)

While we can set $\gamma = t\ C$, β **cannot be both** g **and** x

What to do?

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Changing the A productions to

$$\begin{array}{lcl} A & \rightarrow & g\ A\ t\ C\ x \\ & \mid & \lambda \end{array}$$

permits sentences with too many gs:

$$A \Rightarrow g\ A\ t\ C\ x$$
$$A \Rightarrow g\ g\ A\ t\ C\ x\ t\ C\ x$$
$$A \Rightarrow g\ g\ \lambda\ t\ p\ x\ t\ p\ x$$

The original grammar permits only one g per A recursion.

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We have to use a different (new) non-terminal on the RHS of the new A rule:

$$\begin{array}{lcl} A & \rightarrow & A\ t\ C\ x \\ A & \rightarrow & g \end{array} \equiv \begin{array}{lcl} A & \rightarrow & g\ Q \\ Q & \rightarrow & Q\ t\ C\ x \\ Q & \rightarrow & \lambda \end{array} \equiv \begin{array}{lcl} A & \rightarrow & g\ Q \\ Q & \rightarrow & t\ C\ x\ Q \\ Q & \rightarrow & \lambda \end{array}$$

This equivilency for Q can be reasoned out with a little bit of thought, but it also falls out of our left-factoring pattern if we bend the rules a smidge and recognize Q can be written as $Q \rightarrow Q\ t\ C\ x\ \lambda$ and letting $\gamma = t\ C\ x$ and $\beta = \lambda$.