First Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

Write a sentence from the language of this grammar. . .

First Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

Write a sentence from the language of this grammar...

What terminals of the grammar ($\Sigma = \{b, c, e, g, h\}$) can a sentential form β begin with?[1]

How is this different than the previous question? **We just want the first terminal**, and we are interested in **all possible sentences** of the language.

[1] Recall a **sentential form** is $S \Rightarrow^* \beta$ and $\beta \in (N \cup \Sigma)^*$

First Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

Write a sentence from the language of this grammar...

What terminals of the grammar ($\Sigma = \{b, c, e, g, h\}$) can a sentential form β begin with?[1]

How is this different than the previous question? **We just want the first terminal**, and we are interested in **all possible sentences** of the language.

Congratulations (I hope), you've just found (most of) the **First Set** of S:

$$First(S) \approx \{b, c, e, g, h\}$$

What is the **First Set** of E?

[1] Recall a **sentential form** is $S \Rightarrow^* \beta$ and $\beta \in (N \cup \Sigma)^*$

First Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

Write a sentence from the language of this grammar...

What terminals of the grammar ($\Sigma = \{b, c, e, g, h\}$) can a sentential form β begin with?[1]

How is this different than the previous question? **We just want the first terminal**, and we are interested in **all possible sentences** of the language.

Congratulations (I hope), you've just found (most of) the **First Set** of S:

$$First(S) \approx \{b, c, e, g, h\}$$

What is the **First Set** of E? $First(E) = \{e\}$

[1]Recall a **sentential form** is $S \Rightarrow^* \beta$ and $\beta \in (N \cup \Sigma)^*$

First Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

$First(E) = \{e\}$

Why isn't c in $First(E)$? Notice that $E \rightarrow \lambda$ and rules 2, 7, 9 and 6 permit

$$
\begin{array}{rlll}
(2) & S & \rightarrow & E\ F\ G\ H\ \$ \\
(7) & S & \rightarrow & \lambda\ F\ G\ H\ \$ \\
(9) & S & \rightarrow & C\ E\ G\ H\ \$ \\
(6\&7) & S & \rightarrow & c\ \lambda\ G\ H\ \$ \\
 & S & \rightarrow & c\ G\ H\ \$
\end{array}
$$

so can't c begin an E?

It's time for a formal definition of $First(\alpha)$...

First Sets of α (Formal Definition)

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

$$First(\alpha) = \{t \in \Sigma_\$ | \alpha \Rightarrow^* t\,\beta\} \qquad \alpha \in N, \quad \beta \in (N \bigcup \Sigma)^*$$

Notice: we aren't interested in **sentential forms**, ie: all the rules containing α in the RHS. $First(\alpha)$ is just the terminals that can **begin** the RHS of a derivation **with α on the LHS**.

We also permit ourselves to say $t \Rightarrow t$, even though t is a terminal (or even \$) and does not exist on the LHS of any production rule; what is easy about `First`(α) when $\alpha \in \Sigma_\$$?)

First Sets of α (Computational Definition)

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

$$First(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in \Sigma_\$ \\ \bigcup_{(\alpha \rightarrow \mathcal{X}_i \beta_i) \in P} First(\mathcal{X}_i \beta_i) & \text{if } \alpha \in N \end{cases}$$

$$First(\alpha_1 \alpha_2 \cdots \alpha_n) = \bigcup_{j=1}^{n} First(\alpha_j) \text{ if } \alpha_i \Rightarrow^* \lambda \text{ for } i = 1, \ldots, j-1$$

$First(\lambda) = \emptyset$. The first set of a terminal is itself, the first set of a non-terminal is the union of the first sets of its production rules' LHSs, and the first set of a sequence is the union of its element's first sets **from left to right** up to and including **the first symbol that cannot derive to** λ.

Notice that first sets are from $\Sigma_\$$, the complete $First(S)$ is actually $\{b, c, e, g, h, \$\}$, because $\{\$\} \subset First(E\,F\,G\,H\,\$) \cup First(H\,\$) = \{h, \$\}$.

Follow Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

What symbols from $\Sigma_\$$ might **follow** the rewrite of $B \rightarrow b$ in a derivation using rule 4?

Follow Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

What symbols from $\Sigma_\$$ might **follow** the rewrite of $B \rightarrow b$ in a derivation using rule 4?

B is in the RHS of only rule 1 (which makes things simpler), **in this case** the “follow set” of $B \equiv First(C\,\$)$ because $C\,\$$ comes after B in rule 1.

$$Follow(B) = First(C\,\$) = \{c, \$\}$$

Follow Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\ C\ \$$
2	$S \rightarrow E\ F\ G\ H\ \$$
3	$S \rightarrow H\ \$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\ E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

A little more nuanced: what is the follow set of E?

Follow Sets of Non-Terminals N

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

A little more nuanced: what is the follow set of E?

E is in the RHS of rules 2 and 9,

- from rule 2, $Follow(E)$ gets a contribution from $First(F\,G\,H\,\$) = \{c, e, g\}$
- from rule 9, $Follow(E)$ gets a contribution from $Follow(F)$, since E is at the end of the rewrite rule for F. Fortuitously, the $Follow(F)$ is straightforward in this case: $Follow(F) = First(G\,H\,\$) = \{g\}$

$$Follow(E) = First(F\,G\,H\,\$) \bigcup Follow(F) = \{c, e, g\}$$

Follow Sets of A (Definition)

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

$$Follow(A) = \{t \in \Sigma_\$ | S \Rightarrow^+ \alpha A\, t\, \beta\}$$

$$A \in N \quad \alpha, \beta \in (N \cup \Sigma)^*$$

In this case, we are interested in **only sentential forms** of the language. (S is on the LHS of the set conditional.)

There is no difference between calculating $Follow(t \in \Sigma)$ vs $Follow(A \in N)$, it turns out we will be interested in only the follow sets of **non-terminals** from the grammar.

Follow Sets of A (Computational Definition)

#	Rules
1	$S \rightarrow B\,C\,\$$
2	$S \rightarrow E\,F\,G\,H\,\$$
3	$S \rightarrow H\,\$$
4	$B \rightarrow b$
5	$C \rightarrow \lambda$
6	$C \rightarrow c$
7	$E \rightarrow \lambda$
8	$E \rightarrow e$
9	$F \rightarrow C\,E$
10	$G \rightarrow g$
11	$H \rightarrow \lambda$
12	$H \rightarrow h$

$$Follow(A) = \{t \in \Sigma_{\$} | S \Rightarrow^{+} \alpha A t \beta\}$$

$A \in N \quad \alpha, \beta \in (N \cup \Sigma)^*$

i. Set $Follow(A) = \emptyset$

ii. For each instance of A in a production $X \rightarrow \alpha A \beta$,

   a. Add $First(\beta)$ to $Follow(A)$

   b. If $\beta \Rightarrow^* \lambda$, add $Follow(X)$ to $Follow(A)$